Measuring social reciprocity

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Social research has been mainly focussed on the individualistic approach, although it ignores the social context within individuals are embedded. This fact may explain why dyadic analysis has been increasingly applied to quantify group interactions. Regarding social reciprocity, the directionality of behavior in social interactions can be measured by the Directional consistency (DC) index [1]. This index only allows quantifying social reciprocity at group level, although social researchers are often interested in measuring social reciprocity at group, dyadic and individual levels. The Skew symmetry index can be used to measure social reciprocity at all levels [2]. A statistical test has been recently proposed to make decisions at group level for both statistics [3].

The Skew symmetry statistic is based on data at hand. That is, any sociomatrix is decomposed into its symmetrical and skew-symmetrical parts in order to quantify social reciprocity. Hence, the maximum value of the statistic depends on the number of recorded behaviors. The purpose of our study is to develop a modified version of the Skew symmetry statistic in which the maximum level of asymmetry could be obtained in any sociomatrix. This means that a standardized measurement should be derived.

Social reciprocity in groups can be represented by a matrix $\mathbf{\Pi}$, where the parameter π_{ij} denotes the probability of '*i* addresses a behavior to *j* in each interaction' Note that the parameters of the matrix $\mathbf{\Pi}$ allow defining a measurement of social reciprocity since they contain the main information to quantify dyadic reciprocity among all pairs of individuals. Hence, an index to measure global reciprocity in groups can be defined as

$$\Phi_r = \frac{tr\left(\mathbf{\Pi}'\mathbf{\Pi}\right) - \min\left(tr\left(\mathbf{\Pi}'\mathbf{\Pi}\right)\right)}{\max\left(tr\left(\mathbf{\Pi}'\mathbf{\Pi}\right)\right) - \min\left(tr\left(\mathbf{\Pi}'\mathbf{\Pi}\right)\right)}, \quad 0 \le \Phi_r \le 1$$

Social researchers do not know the value of Φ_r since they collect empirical data. Then, an estimator of asymmetry in social relations is required to obtain some information about social reciprocity. An estimator of the index Φ_r can be defined as follows:

$$\hat{\Phi}_r = \frac{\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} x_{ij}^2/c_{ij}^2 - \min\left(\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} x_{ij}^2/c_{ij}^2\right)}{\max\left(\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} x_{ij}^2/c_{ij}^2\right) - \min\left(\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq 1}}^{n} x_{ij}^2/c_{ij}^2\right)} = \frac{\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} x_{ij}^2/c_{ij}^2 - m}{\max\left(\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} x_{ij}^2/c_{ij}^2\right) - m}, \quad 0 \le \hat{\Phi}_r \le 1$$

where x_{ij}, c_{ij} and n respectively denote the number of behaviors that individual i addresses to j, the amount of interactions in each dyad and the quantity of individuals in groups. It can be proven that the expected value and standard error of the statistic equal

$$E\left[\hat{\Phi}_r\right] = \frac{2\left(\sum_{i=1}^{n}\sum_{j=i+1}^{n} 1/c_{ij} + \sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} \frac{\pi_{ij}^2\left(c_{ij}-1\right)}{c_{ij}} - m\right)}{n\left(n-1\right)-2m}$$

$$\sigma\left(\hat{\Phi}_r\right) = \frac{2\sqrt{\sum_{i=1}^{n}\sum_{j=i+1}^{n} \frac{q_{ij}+q_{ji}+2s_{ij}}{c_{ij}^4}}}{n\left(n-1\right)-2m}$$

$$q_{ij}+q_{ji}+2s_{ij} = 4c_{ij}\left(\pi_{ij}-7\pi_{ij}^2+12\pi_{ij}^3-6\pi_{ij}^4\right)+8c_{ij}^2\left(-\pi_{ij}+6\pi_{ij}^2-10\pi_{ij}^3+5\pi_{ij}^4\right)+ 4c_{ij}^3\left(\pi_{ij}-5\pi_{ij}^2+8\pi_{ij}^3-4\pi_{ij}^4\right)$$

The expression for computing the $\hat{\Phi}_r$ statistic can be rewritten as

$$\hat{\Phi}_r = \frac{\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} x_{ij}^2/c_{ij}^2 - m}{n\left(n-1\right)/2-m} = \sum_{i=1}^{n} \frac{\sum_{\substack{j=1\\ j\neq i}}^{n}\left(x_{ij}^2/c_{ij}^2-\frac{1}{4}\right) - \frac{1}{2}\sum_{\substack{j=i+1\\ c_{ij}\ odd}}^{n}\frac{1}{c_{ij}^2}}{n\left(n-1\right)/2-m} = \sum_{i=1}^{n}\hat{\phi}_i$$

Note that this expression enables to know each individual's contribution to asymmetry in social interactions. The expression for $\hat{\Phi}_r$ can also be written as follows:

$$\hat{\Phi}_r = \sum_{i=1}^{n} \hat{\phi}_i = \sum_{i=1}^{n}\sum_{j=1}^{n} \hat{\phi}_{ij} = \sum_{i=1}^{n}\sum_{j=i+1}^{n} \left(\hat{\phi}_{ij} + \hat{\phi}_{ji}\right) = \sum_{i=1}^{n}\sum_{j=i+1}^{n} \hat{\Phi}_{ij}$$

where $\hat{\Phi}_{ij}$ is a dyadic measurement.

To sum up, this study mainly deals with a new statistic for measuring social reciprocity at global level. Note that the expected value and standard error have been derived for this global measurement of social reciprocity. It allows social researchers to make suitable comparisons between empirical and expected statistic values under specific null hypotheses, often the null hypothesis corresponding to complete reciprocation. A Monte Carlo test is suggested to take statistical decisions regarding null hypotheses since the exact sampling distribution for the global statistic remains unknown. Additionally, the technique also enables social researchers to identify those dyads and individuals that contribute more to the lack of social reciprocity in groups.

References

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